the equations describing the shock transition from state 0 to 1;

$$P_1 = \rho_0 D(D - q_1) = \rho_0 D^2 - \rho_0 D q_1, \tag{7}$$

$$q_1 = (\rho_0 D^2 - P_1)/\rho_0 D. \tag{8}$$

From Eqs. (6), (5), and (8),

$$dq/q_1 = -(A/A_0\rho_0 D)(dP/q_1),$$

$$\frac{q}{q_1} = -\left[\frac{1}{\rho_0 D^2 - P_1}\right]\int_{P_1}^{P}\frac{A}{A_0}\,dP + 1.$$

Integrating by parts

$$\frac{q}{q_1} = -\left[\frac{1}{\rho_0 D^2 - P_1}\right]\left\{\left[\frac{A}{A_0}P\right]_{P_1}^{P} - \int_{A_0}^{A}\frac{P}{A_0}dA\right\} + 1. \tag{9}$$

Noting that

$$\int P\,dA = m(V_p)^2, \tag{10}$$

when energy dissipation in the plates is neglected (m is the mass per unit area of each plate), and taking

$$A/A_0 = 1 + (2x/A_0), \tag{11}$$

approximations which are valid for small values of the angle of deflection of the walls, θ, we obtain

$$q/q_1 = [1/(\rho_0 D^2 - P_1)]$$
$$\cdot[P_1 - P(1 + 2x/A_0) + mV_p^2/A_0] + 1. \tag{12}$$

From Eq. (5):

$$1/\rho = V = Aq/A_0\rho_0 D = (q_1/\rho_0 D)(A/A_0)(q/q_1)$$
$$= [(\rho_0 D^2 - P_1)/(\rho_0 D)^2](1 + 2x/A_0)(q/q_1). \tag{13}$$

The time t required for material to move from the detonation front ($S = 0$) to a distance S behind the detonation is

$$t = \int_0^S (1/q)\,dS. \tag{14}$$

Another quantity, $(dP/d\rho)^{\frac{1}{2}}$, obtainable from the analysis is of particular interest because it is influenced in opposite ways by expansion and by the energy release from chemical reaction. Because of this it serves as a useful measure of the reaction zone of the detonation. The rather complex effects of area increase and chemical reaction may be obtained from Eq. (5) in differential form and from Eq. (6);

$$\frac{q}{\rho}\frac{d\rho}{dq} = \frac{q^2}{dP/d\rho}, \tag{15}$$

$$\frac{q}{A}\frac{dA}{dq} = \frac{q^2}{dP/d\rho} - 1. \tag{16}$$

First, consider the special case when the reaction rate is zero and only lateral expansion is occurring. Then $dP/d\rho = c^2$, so that

$$(q/\rho)(d\rho/dq) = -q^2/c^2, \tag{17}$$

$$(q/A)(dA/dq) = (q^2/c^2) - 1. \tag{18}$$

For subsonic flow, an increase in area is associated with a decrease in flow velocity and an increase in pressure and density; for supersonic flow, the reverse is true.

The effect of an exothermic chemical reaction can be obtained by assuming first that the pressure P is held constant. In this case the velocity q is also constant. Therefore, the area A must be increased to accommodate the increased volume produced by the chemical reaction. If the area A is then adjusted back to its original value with no further chemical reaction, then from Eqs. (17) and (18) the flow velocity increases and the density and pressure decrease in subsonic flow; again the reverse is true in supersonic flow. Chemical reaction and lateral expansion thus have opposite effects on the flow variables; furthermore, the effects of both reaction and expansion are reversed as the flow passes through the sonic or Chapman–Jouguet surface.

Immediately behind the detonation front before appreciable acceleration of the confining plates has occurred,

$$dA/dS = 0, \qquad dP/d\rho = q^2. \tag{19}$$

The behavior of $(dP/d\rho)^{\frac{1}{2}}$ through the detonation is indicated schematically in Fig. 6. We first consider the idealized case (a) of a reaction zone with no lateral expansion until reaction is completed, followed by a zone of lateral expansion with no further reaction occuring. In the reaction zone the flow is subsonic; q rises to meet the (usually) falling c curve at the Chapman–Jouguet state where reaction is complete. In this zone $(dP/d\rho)^{\frac{1}{2}}$ coincides with the q curve [Eq. (19)]. In the expansion zone the flow is supersonic, q continues to increase, and P, ρ, and c decrease. In this zone $(dP/d\rho)^{\frac{1}{2}}$ coincides with c. Thus, $(dP/d\rho)^{\frac{1}{2}}$ reaches its maximum value at the Chapman–Jouguet state where reaction is complete. Considering now the actual case in which reaction and expansion are occurring simultaneously, we note in Fig. 6(b) that the q and c curves again cross at the Chapman–Jouguet state or sonic point, where reaction is now only partial. The quantity $(dP/d\rho)^{\frac{1}{2}}$ initially coincides with q immediately behind the front, but later falls below the q curve because of lateral expansion. Eventually, as reaction

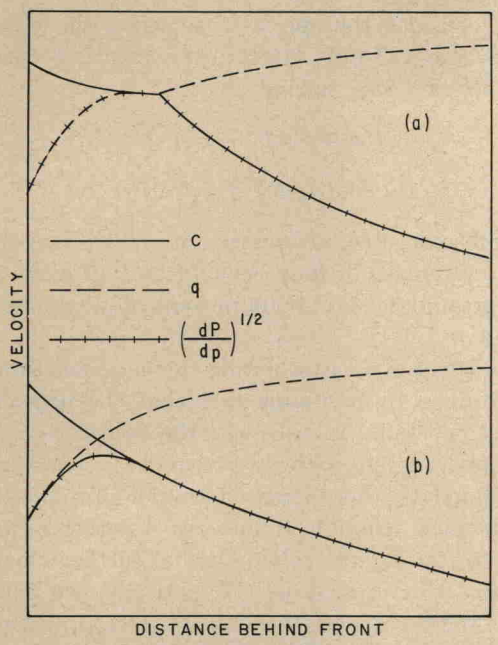

FIG. 6. Schematic variations of sound speed (c), flow rate (q), and $(dP/d\rho)^{\frac{1}{2}}$ with distance behind the detonation front for two detonation cases: (a) case of reaction zone with no lateral expansion until completion of reaction; and (b) case of overlap of reaction and expansion regions.

proceeds to completion downstream from the Chapman–Jouguet state, $(dP/d\rho)^{\frac{1}{2}}$ approaches the falling c curve. Thus, $(dP/d\rho)^{\frac{1}{2}}$ goes through a maximum which will generally lie near the sonic point. The exact location of the sonic point cannot be obtained from the experimental data since the sound velocity cannot be determined (unless the equation of state of the mixture is known). From the experimental approach, the maximum in $(dP/d\rho)^{\frac{1}{2}}$ appears to be a reasonable measure of the length of the effective reaction zone.

One would expect that usually the pressure will fall monotonically behind the detonation front, since lateral expansion gradually increases from an initial zero value; whereas, reaction is usually high behind the front. In this case, reaction dominates lateral expansion in the subsonic zone, while the reverse is true in the supersonic zone. However, the reaction rate may be very low for an appreciable distance behind the front (in an "induction" zone), in which case the pressure may rise in the subsonic zone because of the dominance of lateral expansion over reaction. Eventually the reaction rate must speed up in order that the sonic velocity may be passed.

Effects of Explosive Thickness and Confinement

The detonation process can be influenced on the one hand by changing some property of the explosive to vary the reaction rate, and on the other hand by changing the thickness of the explosive and the degree of confinement, which affect the rate of lateral expansion. In the experiments described in this paper, the reaction rate of the explosive was varied by changing the particle size of one of the ingredients (AN) while keeping the composition (80% AN and 20% TNT) and the density fixed. If the thicknesses of the explosive and of the confining plates are fixed, the relative rates of chemical reaction and lateral expansion will be changed as the particle size is changed. The extent of chemical reaction occurring ahead of the Chapman–Jouguet point will be changed, with a consequent change in the detonation velocity. The altered conditions of pressure and temperature as a consequence of the change in detonation velocity will influence the rate of reaction. Thus, a change in particle size produces a complex effect on the measured reaction time. The direct effect of particle size on reaction rate can be obtained if the thickness of the explosive layer and the degree of confinement can be modified in such a way that the relative rates of reaction and lateral expansion remain unchanged. If the change in particle size changes the time scale of the reaction without changing the form of the reaction rate curve then a change in the thicknesses of explosive and confining plates by the same scale factor by which the time scale of the reaction has been changed will not change the relation between reaction and lateral expansion. Consequently, the detonation velocity will be unchanged. The ratio of measured reaction times should equal the scale factor.

If, on the other hand, the change in particle size changes the form of the reaction rate then a change of the thicknesses of both explosive and confining plates by the same factor will not maintain an unchanged ratio of reaction rate to lateral expansion throughout the reaction zone. For a given thickness ratio, the ratio of particle sizes which keeps the detonation velocity unchanged will differ from the particle size ratio which gives a ratio of reaction times equal to the thickness ratio. In discussing the present experimental results, we designate as "scale shots" those pairs of shots made with different particle sizes and different thickness for which the detonation velocity is substantially unchanged. In every case, the thickness of the confining plates was changed by the same factor as the thickness of the explosive.

In indirect methods of deriving the reaction time from measurements of the effects of explosive diameter on the detonation velocity, for example, by